

Appendix B

Elementary School Sample Lesson: An East Asian Approach

Lesson One, “Introduction to Adding—The Adding-on Concept,” is one of five lessons from Unit Five, “Addition with Sums Up to Ten,” from “M-Math,” an elementary school mathematics curriculum being developed by the University of Michigan and based on an East Asian curricular approach. This is just one lesson in a series covering five strands of mathematics concepts: number sense, operation and computation, measurement, geometry, and quantitative relations. These strands are introduced sequentially through in-depth discussion of each of the systemic laid-out mathematics concepts. The M-Math project was directed by Dr. Shin-ying Lee at the Center for Human Growth and Development, University of Michigan, with the help of many colleagues and teachers. It is an effort, based on what has been learned from the East Asian approach, to adapt and develop mathematics lessons to be taught in American classrooms. The project is supported by the Pew Charitable Trusts and the University of Michigan.

Unit Five

Addition with Sums Up to Ten

Lesson 1	Introduction to Addition—The Adding-on Concept
Lesson 2	The Combining Concept of Addition
Lesson 3	Both Types of Addition Problems
Lesson 4	Addition with Zero
Lesson 5	Vertical Addition

Introduction to the Unit

Up to this point, students should have learned and mastered the concept of numbers through ten. In this unit students will be introduced to two concepts of addition. The first is the *adding-on* concept, which involves gradually adding objects to an original group and finding the resulting quantity. The second type is the *combining* concept, or adding two separate groups together and finding the combined quantity. These concepts illustrate two related yet distinct situations in which addition is used.

Students will also learn how to write addition equations and to recognize and use key mathematical terms related to addition (e.g., *add*, *more*, *total*, *altogether*) in this unit. To facilitate students’ understanding of the relationship between the addition concepts and the written equation, all addition problems are presented in the form of addition “stories.” Moreover, students are expected to come up with their own story problems for each type of addition concept. At the end of this unit, students will learn how zero can be incorporated into addition equations as well.

Goals of the Unit

At the end of this unit, students should be able to:

- Understand the meaning of the two approaches to addition introduced in this unit:
 1. The adding-on concept
 2. The combining concept
- Apply addition to real-life examples in which the sum is ten or less.
- Learn and use the terms and symbols related to addition, such as *add*, *addition*, *equation*, $+$, and $=$.
- Read and write addition equations.
- Understand the meaning of addition equations that involve zero.

Essential Mathematical Concepts in Unit Five

The mathematics concepts presented in this unit are the meaning and applications of addition, important points to keep in mind when teaching addition, important aspects of equations, and addition involving zero.

The Meaning and Applications of Addition

Although addition is a straightforward mathematical operation, first grade students often demonstrate difficulties in its application. For example, students sometimes use addition in situations in which they should be using subtraction or vice versa. To minimize these kinds of errors in students' work, teachers need to provide opportunities for students to learn addition in the context of real-life situations.

Addition is generally used in the following four situations. Students will learn the first two concepts in the first grade. The last two will be introduced in the second grade.

1. *Adding-on concept.* To find the final quantity of a group after additional members have been added to the original group. For example: *There are 5 fish in a tank. John put 2 more fish in the tank. How many fish are in the tank altogether?*
2. *Combining concept.* To find the total quantity of two simultaneously existing groups by combining them together. For example: *There are 5 children playing on a slide. There are 2 children playing on the swings. How many children are playing on the playground altogether?*
3. *Comparing concept.* To find the quantity of a group by stating how many more objects of that group exist compared with the number of objects in a reference group of a known quantity. For example: *There is a pile of red and white balls. There are 5 red balls in the pile. There are 2 more white balls than red balls. How many white balls are there?*

4. *Reverse subtracting concept.* To find the original quantity by adding together the final quantity and the quantity that was subtracted. For example: *John used \$5 to buy a toy. He has \$2 left. How much money did John have in the beginning?*

Important Points for Teaching Addition

When teaching addition, teachers must keep the following points in mind:

- Students need to be able to count accurately before addition is introduced.
- It is important for students to develop the concept of quantity and to use concrete situations when thinking of addition. It is, therefore, essential to use real-life examples (e.g., concrete pictures or word problems that depict the first two types of addition concepts listed in the preceding section and that are related to the students' daily experience) and to use manipulatives to facilitate students' understanding of addition. Students should learn to use terms such as *more*, *combine*, *join together*, *altogether*, *put together*, and *in total* when discussing addition problems.
- It is also important for students to learn how to represent a concrete situation in abstract terms (i.e., in equation form). By the end of this unit, students should be able to explain how to solve the addition problem and write an equation by using symbols such as + and = to represent the situation.
- Students should learn to express their thinking process from concrete to abstract when solving the problems. They can proceed in the following way:

*Problem: There are 3 fish in a tank. John put 1 more fish in the tank.
How many fish are in the tank altogether?*

Thinking processes to solve the problem follow:

1. In the beginning there are 3 fish, and 1 more is added, making a total of 4 fish altogether.
2. 3 fish and/plus 1 fish are 4 fish.
3. 3 and/plus 1 are 4.
4. $3 + 1 = 4$.

These processes of instructions illustrate the adding-on concept of addition, an increase in group size.

Important Aspects of Equations

In this unit students are exposed for the first time to mathematical equations. It is very important to have students understand what the equations represent. Students should learn that an equation represents not only the numerical calculation but also a simpler way of stating a situation by using symbols instead of words. Teachers should let students explain the equations by using pictures, making up their own story problems, or drawing pictures to correspond to a particular equation. The emphasis on verbal expression of mathematical equations is an important way to ensure that the students thoroughly understand them.

When writing equations, students must know the following points about the components of equations:

1. Equations are composed of symbols that represent *quantities* or *objects*, such as numbers.
2. Equations are composed of symbols that represent the *mathematical operations* to be performed, such as $+$, $-$, \times , \div .
3. Equations are composed of symbols that represent the *equality or inequality* of single terms or of terms that involve some kind of mathematical operation. Such symbols include $=$, $<$, $>$.

A complete equation has to have all three components. Make sure that students do not write incomplete equations, such as $3 + 2 = \underline{\hspace{1cm}}$.

Addition Involving Zero

Students have already learned that *no quantity* is represented by zero. Using this fact, students will learn how to calculate equations that involve zero (e.g., $2 + 0 = 2$, $0 + 3 = 3$, and $0 + 0 = 0$). It may not be easy for students to understand the meaning of these equations if verbal explanations alone are used. Therefore, it is important to have students engage in activities, such as the “toss-a-ball-in-the-box” game, that will show them how to make the equation correspond with actual events. As the students play the “toss-a-ball-in-the-box” game and try to figure out their total score after two trials, they will have an opportunity to understand the setting for addition of zero; for example, $2 + 0$.

Make sure that students understand that the end product (sum) does not increase when zero is added.

Lesson One **Introduction to Addition— The Adding-on Concept**

- Lesson Goals**
1. To introduce the concept of addition in which additional members are added to a main group
 2. To understand how to express the idea of addition with an equation

Materials	Teacher	Students
	<ul style="list-style-type: none"> • Poster (fish tank) • Poster (bird) • Round magnets 	<ul style="list-style-type: none"> • Workbook (5.1, 5.2, 5.3) • Homework (5-1)

- Lesson Plan** **Activity One**
- Introduce the idea of adding objects to a main group and then finding the resulting quantity.
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- Steps**
1. Put the fish-tank poster on the board. Have the students make up stories while looking at the picture on the board and talk freely about the scene. After the students have offered their ideas, summarize what they have said.
 2. Make sure the students understand that one fish is being added to an original group of three fish. Finally, have them understand that the main question is, How many fish are there altogether?



Points to Be Aware Of

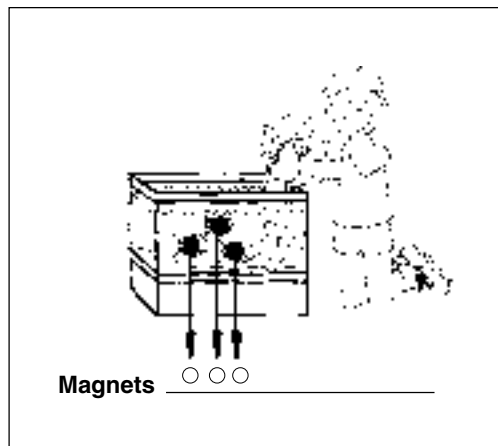
Always restate the students' relevant comments in a clear, concise way, leading the students to build up the understanding of the adding-on concept.

Activity Two

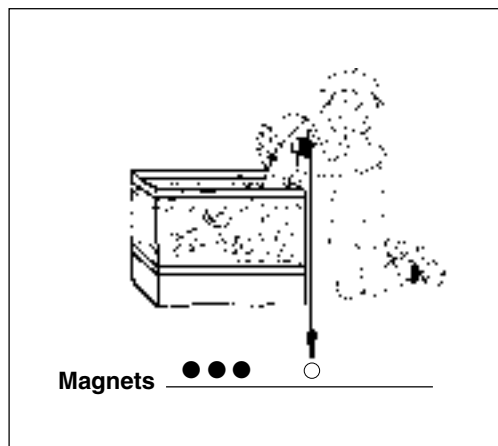
Use manipulatives to reinforce the adding-on concept of addition.

Steps

1. Now replace the fish with magnets. Begin with the three fish in the tank. Tell the class, *"First there are 3 fish. I will use 1 magnet to represent 1 fish."* Place one magnet on each one of the three fish in the picture. Then move the magnets underneath the picture.



2. *"Then, 1 fish was added."* Place another magnet on the fish being added and then move that magnet underneath the picture.



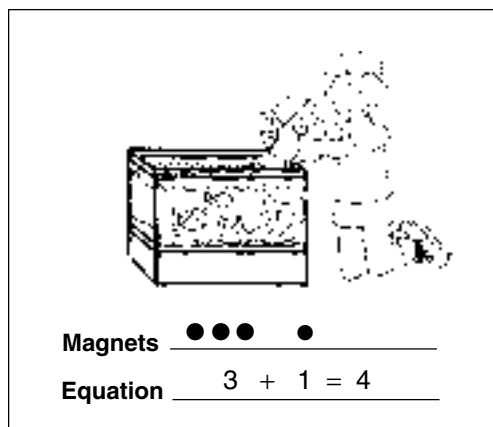
3. *"How many fish are there altogether? Let's count the magnets and find out."* Count as a class, *"1, 2, 3, 4; there are 4 fish altogether."*
4. Repeat the problem straight through aloud. *"First there were 3 fish."* Put the magnets on each of the fish and then move the three magnets down. *"Then 1 fish was added."* Put one magnet on the fish and then move it down. *"There are 1, 2, 3, 4 (counting the magnets), 4 fish altogether."*

Activity Three

Understand how to express the idea of addition with an equation
($3 + 1 = 4$).

Steps

1. Tell the class: *"We can write what happened in this picture by using numbers. Here is how to write this problem. There are 3 fish in the beginning, so we write a 3."* (Write the 3 on the board.) *"We are adding some fish, so we write the plus sign."* (Write a + next to the 3.) *"How many fish did we add? We added 1 fish. So we write a 1 on the other side of the plus sign."*
2. Go over this part of the equation again if the students seem confused.
"Now, the question asked is, How many fish do we have altogether? To show the processes we are using to figure out how many fish there are altogether, we write an equal sign." (Write an = sign.) *"Then, we write the total number of fish we have altogether, 4."* (Write a 4.)
"This is an equation. It tells us how many we have to start with, how many were added, and how many we have altogether at the end."
3. Call on a couple of students and ask them to write the equation and explain to the class the meanings that go with it. They can talk about each part of the equation as they write it.
4. Have the students turn to Workbook page 5.1 and write the equation, as shown in the illustration below.



Points to Be Aware Of

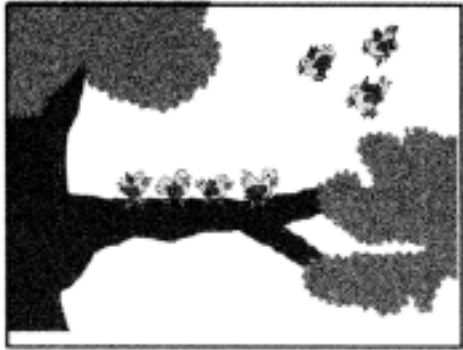
It is important to spend time teaching students how to read and express equations. Make sure that the students understand that an equation, through the use of numbers and symbols, can concisely represent the meaning of the situations.

Activity Four

Review the adding-on concept and introduce the drawing-circles method.

Steps

1. Display the bird poster. Ask the whole class to discuss what is happening in the picture.
2. Review briefly the entire procedure (which is the same as that for Activity Three) with the entire class and then call on several students to go through the process:
 - Place magnets on the first group of birds and move the magnets down. Place magnets on the group being added and move those magnets down. (Move the three magnets into the group of four magnets.)
 - Count the total number of magnets aloud.
 - Ask the student, as the student moves the magnets, to describe what he or she is doing and what it means.
3. Ask a different student to come up and write the equation on the board. Have the student explain the meaning of each component of the equation. Have other students elaborate on the meaning of the equation if it is not clearly explained.
4. Display Workbook page 5.2, shown below, on the overhead projector. Explain to the students that this time, instead of using magnets, they will draw circles to represent the magnets. (They can use this method when no magnets are available.) *“First there are 4 birds on the tree branch, so we draw 4 circles to represent the birds on the branch. Then 3 birds fly in, so we draw another 3 circles to represent the birds flying in.”*



(Draw circles.) ○ ○ ○ ○ ○ ○ ○

(Equation) + =

Activity Four (Continued)

5. “*Now, how many birds do we have altogether?*” Have the entire class count and call out the answer.
6. Have the students turn to their Workbook page 5.2. Give the students time to draw their circles and fill in the blanks in the equation part. As the students work, walk around the room to see how they are doing. When they have finished, have two students with different answers come up to the board and write their equations. Have each student explain how he or she came up with the equation. Discuss these equations with the class.
7. Have the students turn to Workbook page 5.3 and work on one problem at a time. After they have finished each problem, ask a student to come to the front of the room, talk about the problem, and write his or her equation for it. Then discuss this equation and the answer. Ask whether any student has any different answer or different explanation to that problem.

Summarize the Lesson

Summarize the adding-on concept and the representation of an equation for the lesson.